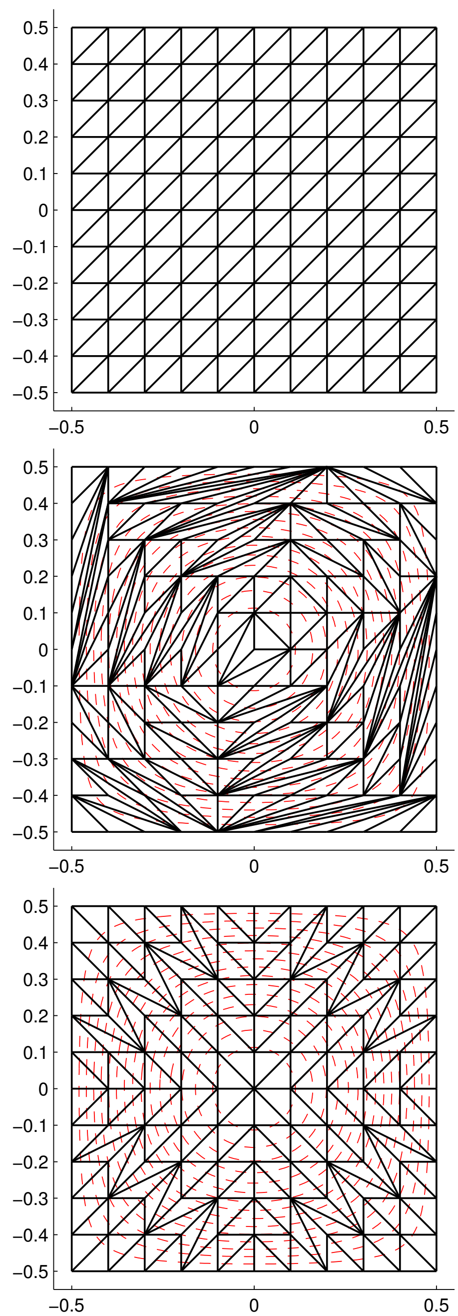
Mathematical Modeling and Analysis

Rezoning of Triangular Meshes in ALE Simulations by Node Reconnection

Pavel Vachal, vachal@t7.lanl.gov
Mikhail J. Shashkov, shashkov@lanl.gov
Rao V. Garimella, rao@lanl.gov

The Arbitrary Lagrangian-Eulerian (ALE) methodology [1] is a promising approach for simulations of complex fluid flows. Various physical applications put various requirements on practical implementation of the particular steps of the ALE framework. Here, we focus on the mesh rezoning part, that is on the issue how to improve the mesh quality in order to keep the computation running by making the next Lagrangian step possible.

There are two ways to rezone the mesh: node repositioning and node reconnection. We believe that an ideal method has to combine both approaches. However, to gain full control about the rezoning process, one first has to understand and manage each of the two techniques separately. Recently, we suggested a set of methods based on mesh repositioning, without changes of connectivity [2] [3]. Here, we follow the other path. Positions of nodes stay fixed and the mesh quality is being improved by changing of the edges between them. For simplicity, we are working on an unstructured triangular mesh, where the process can be simply described by a sequence of edge "swaps". Consider two neighboring triangular cells, forming an quadrilateral, split by the common edge of both cells. By edge swapping, we mean replacement of this edge by the other diagonal of the quadrilateral. Our mesh rezoning algorithm is as follows: Go in loop through all interior edges. For each edge, compare the quality of the pair of adjacent cells with quality of the alternative pair, created by swapping of the current edge. If the new cells are better, keep the edge swapped. Continue with the next edge. Stop if there is no change during the whole loop.



Top: The original mesh. Center: Result based on piecewise constant reconstruction of discrete values $f_i = \cos(\pi x_i)\cos(\pi y_i)$ and with numerically approximated derivatives. Bottom: Result based on piecewise linear reconstruction of f_i and with numerical derivatives. Isolines of (smooth) underlying function f shown in dashed red.

Rezoning of Triangular Meshes in ALE Simulations by Node Reconnection

The basic question is how to measure the cell quality. We tested and combined a wide set of criteria, from pure geometrical ones (based on condition number for particular vertices, average cell areas, etc.) to those more physics related. Here we present some of the latter ones, since our main objective is to develop a robust and universal mesh rezoning method for ALE simulations. The experience shows, that geometrical changes of the mesh should be closely related to behavior of the simulated physical system under study to achieve best results. We seek to meet this requirement by utilizing the discrete values of a selected state variable, for example density in fluid dynamics or temperature in plasma physics. This will allow us to test the whole method in ALE simulations of many phenomena, such as Rayleigh-Taylor instability or laser-plasma interactions.

Let us first suppose, that the selected state variable is described by a smooth function $f(x,y)$. In each cell, we approximate it by a constant or linear reconstruction. The quality measure will then be the integral of the reconstruction error over the whole cell. This error is approximated by square of the next terms in Taylor expansion, particularly terms with f_x, f_y for piecewise constant reconstruction or f_{xy}, f_{xx} and f_{yy} for the piecewise linear case. However, in practical ALE simulations, a physical quantity will be always described by a set of discrete values rather than by a smooth, differentiable function. In this case, one has to approximate numerically also the derivatives. We first tested an approach from [4] utilizing Green's theorem and then suggested another method, based on integration of forward differences and minimization of a suitable functional.

The idea of proposed rezoning method was first implemented with analytical underlying function $f(x,y)$. In all tests, the general behavior was the same: as expected, all cells tend to align either parallel or perpendicular to isolines of the function. This is exactly what one needs in ALE simulations: the mesh geometry automatically keeps track with some variable of the simulated physical process. Aligning of cells is more dramatic in the case of piecewise constant reconstruction, since the difference to the underlying function (i.e. the L_2 error of interpolation) is bigger. With the piecewise linear reconstruction, the effect is still strong enough and moreover the cells keep a satisfactory quality. Finally, practical applicability of the method was proven by an implementation with numerically approximated derivatives, utilizing a discrete underlying function. Results of this latter approach are in good correspondence with the analytical case.

There is no doubt that a good mesh rezoning strategy can be achieved as a combination of node reconnection with node repositioning (see e.g. [2] [3]), but we believe that it is first necessary to understand and manage both approaches in detail in order to develop their fully controlled combination. Furthermore, the developed edge-swapping technique will be used by other members of the T-7 group in their research involving solution of elliptic problems.

Acknowledgements

Los Alamos Report LA-UR-04-3728. Funded by the Department of Energy under contracts W-7405-ENG-36 and the DOE Office of Science Advanced Computing Research (ASCR) program in Applied Mathematical Sciences. Funded by the ASCI program in the Department of Energy under contracts W-7405-ENG-36. P.V. was partly supported by FRVS grant 2087/2004, Czech Grant Agency grant GACR 202/03/H162 and Czech Tech. University internal grant CTU 0411014.

References

[1] C. Hirt, A. Amsden, and J. Cook. An arbitrary Lagrangian-Eulerian computing method for all flow speeds. *J. Comp. Phys.*, 14(3):227–253, 1974.

[2] P. Váchal, M. Shashkov, R. Garimella, and R. Loubère. Untangling of meshes in ALE simulations. Presented at 7th USNCCM, July 27-31, 2003, Albuquerque, NM. Tech. Rep. LA-UR-03-6100, Los Alamos National Laboratory, 2003.

[3] P. Váchal, R. Garimella, and M. Shashkov. Untangling of 2D meshes in ALE simulations. *J. Comp. Phys.*, 196(2):627–644, 2004.

[4] M. Shashkov. *Conservative Finite-Difference Methods on General Grids.* CRC Press, Boca Raton, 1995.